Semiclassical tunneling in the BEC dimer

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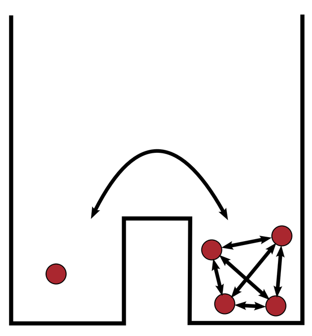
The Big Question

What's the simplest way to think about dynamical tunneling in an optical lattice?

System

The BEC dimer consists of two coupled wells containing N condensed bosons. Neglecting states other than the lowest in each well, we get the Bose-Hubbard dimer:



$$\hat{H} = -J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + \frac{U}{2}(\hat{n}_1(\hat{n}_1 - 1) + \hat{n}_2(\hat{n}_2 - 1)),$$

where $\hat{a}_i$ is the annihilation operator for a boson in well i and $\hat{n}_i \equiv \hat{a}_i^\dagger \hat{a}_i$ is the number operator.

Mean-field model

A mean-field approximation is obtained by replacing operators with functions [1]. Define,

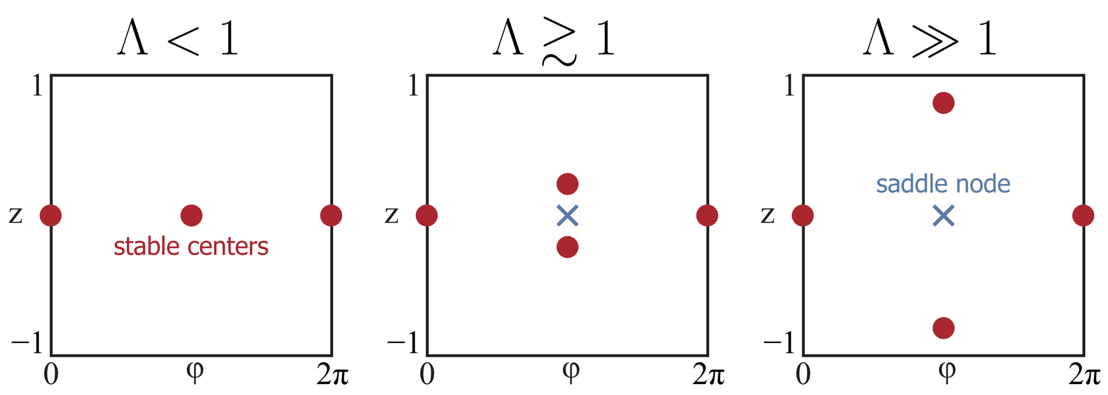
$$z = \frac{\langle n_1 - n_2 \rangle}{\langle n_1 + n_2 \rangle} \quad \text{Population imbalance}$$

$$\exp(\imath\phi) = \frac{\langle a_1^\dagger a_2 \rangle}{\|\langle a_1^\dagger a_2 \rangle\|} \quad \text{Relative phase}$$

The evolution of z and ϕ is governed by the Hamiltonian,

$$H_{\mathrm{MF}} = \frac{\Lambda z^2}{2} - \sqrt{1 - z^2}\cos\phi$$

where $\Lambda = UN/2J$ captures the strength of particle interactions. The mean-field model exhibits a bifurcation at $\Lambda = 1$:



The two centers at $|z| > 0$ are the *self-trapping points*. Experiments confirm the predictions of the mean-field model, at least for relatively short times [2, 3].

Self-Trapping & Tunneling

self-trapping In the mean-field model, a system initially in a coherent state sufficiently near one of the stable fixed points remains in its neighborhood forever.

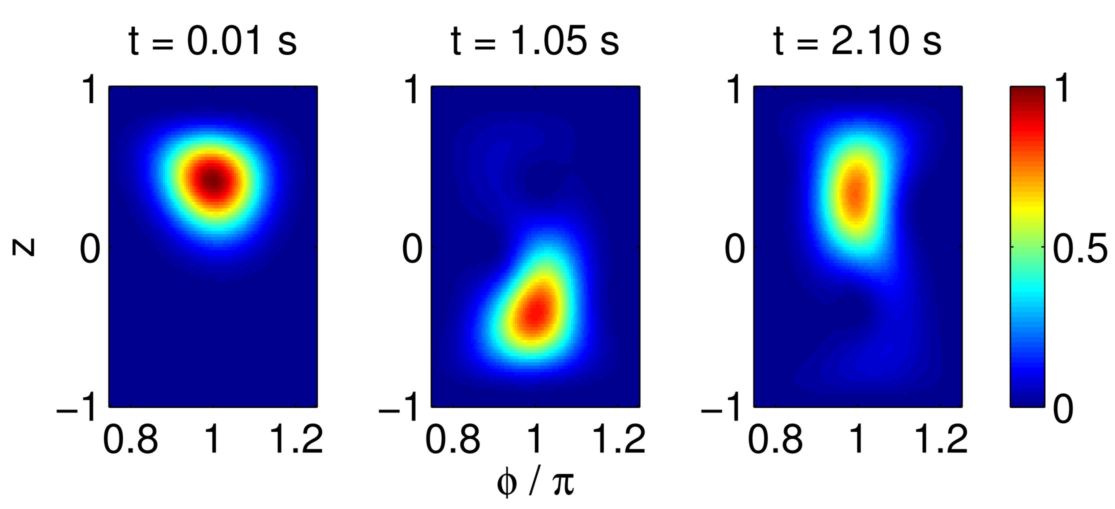
tunneling In the full quantum treatment, tunneling takes place between the self-trapping points.

To visualize the tunneling, define the Husimi function of a state $|\psi\rangle$:

$$Q_\psi = |\langle \psi | z, \phi \rangle|^2,$$

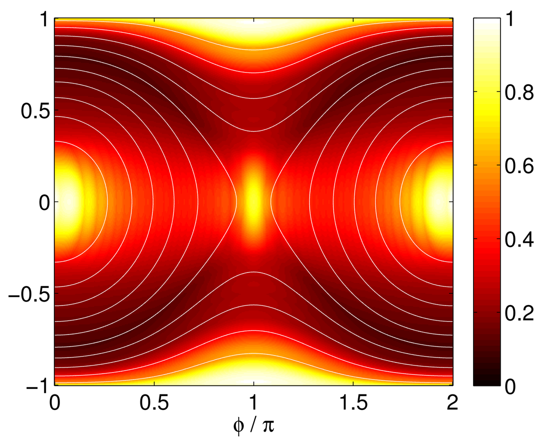
where $|z, \phi\rangle$ is a coherent state [4].

Below see Q_ψ over time for a coherent initial state at a self-trapping fixed point ($N = 40$, $\Lambda = 1.1$ and $J = 10\,\hbar/\mathrm{s}$):



Dimensional reduction

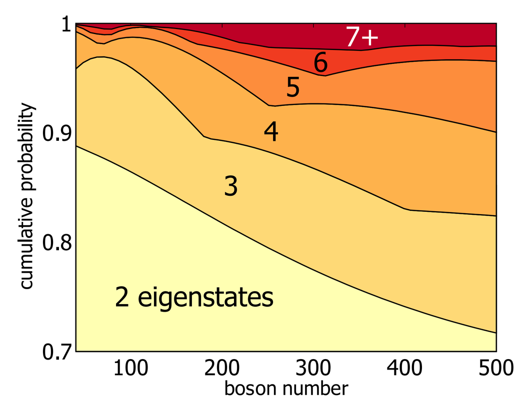
Let $|E_0(z,\phi)\rangle$ and $|E_1(z,\phi)\rangle$ be the two most probable energy eigenstates for a given coherent state $|z,\phi\rangle$.



Their contribution $\sum_{i=0,1} |\langle E_i \mid z, \phi \rangle|^2$ to the total probability weight of $|z,\phi\rangle$ is shown on the left (for $N = 40$, $\Lambda = 5$).

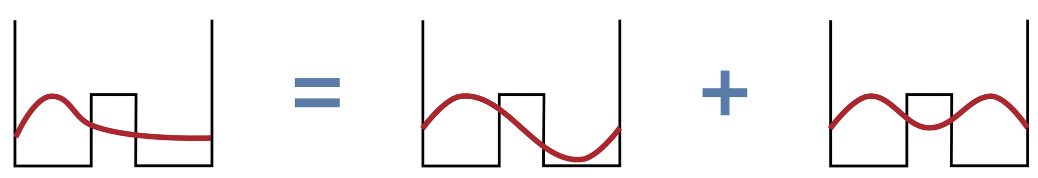
Near the self-trapping fixed points, only a few energy eigenstates contribute to the dynamics of the coherent state.

But note that as N increases, the system becomes "less discrete" and more states are needed for an effective approximation.



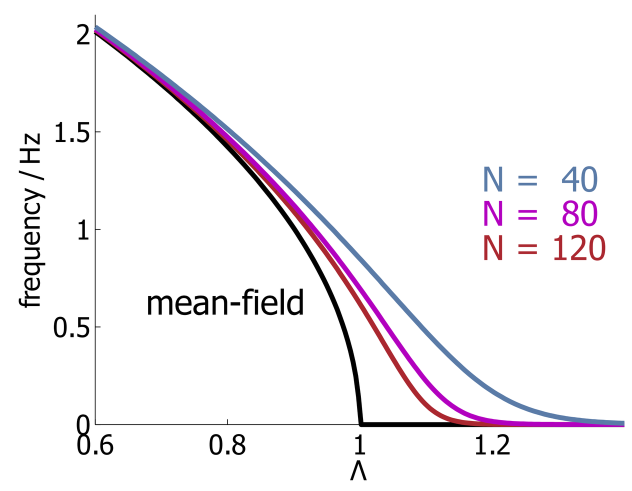
Two-state model

Coherent state ≈ symmetric + antisymmetric states.



This ansatz produces very accurate predictions. The energy difference between the two eigenstates is very nearly equal to the dominant frequency in the power spectrum obtained by integrating the Schrödinger equation numerically.

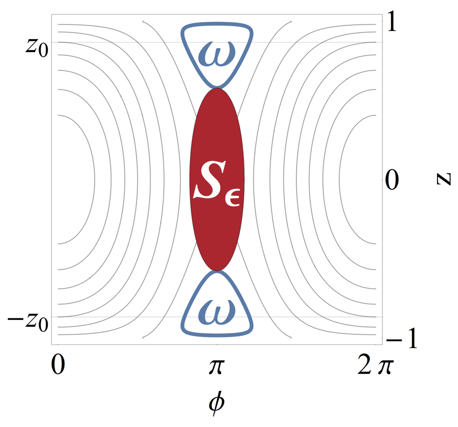
On the right, predicted tunneling frequencies for various N, with $J = 10\,\hbar/\mathrm{s}$.



The mean-field limit is approached as N is increased, but how quickly? What determines the Λ dependence?

Semiclassical quantization

We extend the approach of [5]; see also [6]. If the splitting due to the tunneling is small, it is,



$$\Delta E \approx \hbar\omega \exp(-\pi S_\epsilon)$$

with ω the classical frequency and S_ϵ the (Euclidean) action associated with the tunneling.

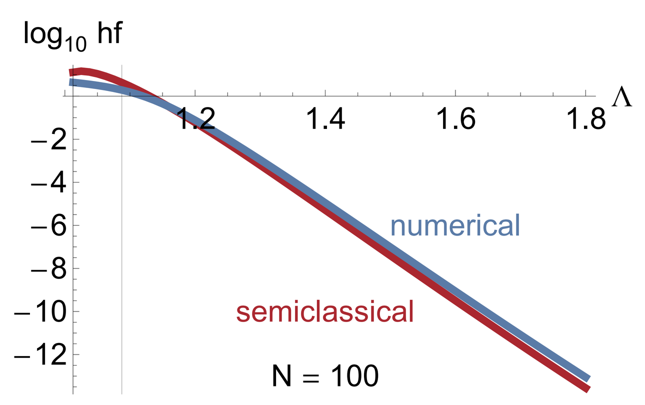
Via asymptotic approximations we find that for the ground state, for large N, the tunneling frequency is,

$$f \approx \frac{\omega}{2\pi^2}\left(\frac{2}{\omega}\exp(-z_0)\right)^{(N+1)(1-e)}$$

where $z_0 = \sqrt{1 - 1/\Lambda^2}$ is the position of the fixed point and,

$$\omega \approx 2\sqrt{\Lambda^2 - 1}$$

$$e \approx \frac{2\Lambda\sqrt{\Lambda^2 - 1}}{(\Lambda - 1)^2(N + 1)}$$

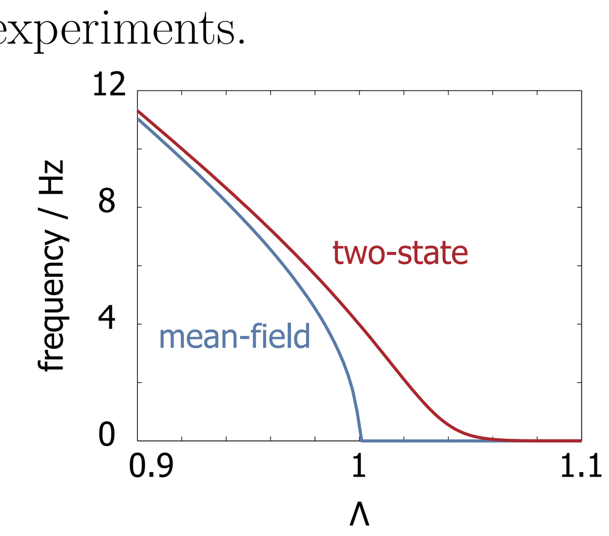


This simple expression works very well, except for $(\Lambda - 1)$ so small that the self-trapping regions have area smaller than $h/2$.

Predictions for experiment

In principle, tunneling between the fixed points should be observable in current experiments.

The tunneling frequency expected in the experiment of [3] ($N = 500$ and $U = 2\pi \times 0.063\,\hbar/\mathrm{s}$) is shown on the right.



But there are challenges:

retention time experimental trapped atom lifetimes are only ~ 100 ms

tilt tunneling frequency will be lowered if the wells' chemical potentials differ [7]

Only a Few States Matter

Sufficiently near the self-trapping fixed point, only two energy eigenstates have significant overlap with the coherent state. These states are symmetric and antisymmetric combinations of localized states, and their energy splitting can be accurately estimated semiclassically.

References

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Acknowledgments

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